LOCAL RELIEF AND THE HEIGHT OF MOUNT OLYMPUS

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Received 26 June 1999; Revised 6 July 1999; Accepted 23 August 1999

ABSTRACT
A three-dimensional assessment of the net volume of rock differentially eroded from below mountain tops to form valleys yields a range-wide constraint on feedback between valley development and the height of mountain peaks. The ‘superelevation’ of mountain peaks potentially attributable to differential removal of material from below peaks in the Olympic Mountains, Washington, was constrained by fitting a smoothed surface to the highest elevation points on a 30 m grid digital elevation model of the range. High elevation areas separate into two primary areas: one centred on Mount Olympus in the core of the range and the other at the eastern end of the range. The largest valleys, and hence areas with the greatest volume of differentially eroded material, surround Mount Olympus. In contrast, the highest mean elevations concentrate in the eastern end of the range. Calculation of the isostatic rebound at Mount Olympus attributable to valley development ranges from 500 to 750 m (21 to 32 per cent of its height) for a 5 to 10 km effective elastic thickness of the crust. Comparison of cross-range trends in mean and maximum elevation reveals that this calculated rebound for Mount Olympus corresponds well with its ‘superelevation’ above the general cross-range trend in mean elevation. It therefore appears that the location of the highest peak in the Olympics is controlled by the deep valleys excavated in the centre of the range.

KEY WORDS: Valley incision; peak uplift; erosion

INTRODUCTION
The height of local peaks is perhaps the most prominent geomorphological aspect of a mountain range, and tectonic and erosional processes generally influence the elevation of mountain peaks over different length scales. Tectonic processes govern the average crustal thickness, spatial variability in rock uplift and the flexural rigidity of the crust, which in turn control the general elevation of a mountain range. Incision of local relief can trigger isostatic uplift which compensates for the net mass of rock eroded to form valleys and thereby further raises mountain peaks (Holmes, 1944). Isostatic rebound can also focus rock uplift in areas of concentrated erosion (Beaumont et al., 1992; Willett et al., 1993; Zeitler et al., 1993). In addition, flexural uplift of mountains can occur by loading of the crust, which can result from volcanic loading by seamounts (McNutt and Menard, 1978), along rift zones or other major topographic discontinuities (Gilchrist and Summerfield, 1990, 1991), and on the margins of depositional basins (Quinlan and Beaumont, 1984; Small and Anderson, 1995). The heights of mountain peaks therefore reflect a combination of external geophysical forcing and spatial variations in erosional and depositional processes.

The potential for elevating mountain peaks through valley deepening was recognized shortly after acceptance of the role of isostasy in maintaining mountain ranges (see Suess (1909) and Bowie (1927) for discussions of the early development of the concept of isostasy) (Figure 1). The idea of raising mountain tops in response to isostatic adjustment to denudation is simple—erosion of crustal rocks decreases the mass of the column of rock above the mantle, which then triggers rock uplift (sensu England and Molnar, 1990) as material at depth moves in to compensate for the mass deficit. Because crustal rocks generally are less dense than deeper rocks, the volume of the compensating rock is less than that of the eroded rock. Hence, a mountain range will wear down evenly if erosion occurs uniformly, as rebound will only refresh a fraction of...
the lost elevation. In contrast, differential erosion that incises deep valleys can raise mountain tops, even though the mean elevation of the landscape decreases (Holmes, 1944). Only through valley deepening or widening can erosional processes contribute to elevating peaks, as material uniformly stripped off of a landscape will result in global lowering.

The relative contribution of valley incision to the height of mountain tops is a constraint on the interpretation of evidence for feedback between late Cenozoic mountain uplift and climate change (Molnar and England, 1990). Evaluation of the specific rise of mountain peaks in response to a particular episode of valley deepening requires knowing both the initial topography and the degree of flexural support due to crustal strength. Unfortunately, range-wide initial conditions are very difficult to know except where undisputed erosion surfaces are preserved, and even in such special cases cosmogenic isotope data can suggest substantial erosion of the presumed palaeosurface (Small and Anderson, 1998). For the general case where one cannot identify an initial surface, it would be useful to be able to constrain the maximum possible contribution of valley development to the height of peaks. The simplest case is that of static isostatic compensation for the net mass of rock that is ‘missing’, or differentially eroded from below mountain peaks.

How much of the elevation of mountain peaks is due to the development of valleys? The idea that incision of deep valleys can elevate mountain peaks has a long history, but the magnitude of this effect has been constrained in only a few studies. Jeffreys (1929, p. 295) considered ‘deepening of the valleys’ to cause only a ‘slight rise of the highest ground’. In contrast, Wager (1933, 1937) proposed that isostatic compensation for the mass of rock carved out of deep valleys was responsible for the great height of Himalayan peaks. Based on Wager’s argument, Holmes (1944) estimated that incision of the Arun River gorge could account for 21 per cent of the height of Mount Everest. Although the maximum uplift attributable to local relief can be calculated from the volume of material missing from between mountain peaks for the case of local isostatic compensation, flexural rigidity of the crust limits actual compensation to less than that predicted for purely local compensation. Taking this into consideration, Montgomery (1994) analysed topographic cross-sections and showed that simple isostatic response to incision of deep valleys could account for at most 5–10 per cent of the elevation of Sierran peaks but up to 30 per cent of the elevation of Himalayan peaks. In a similar generalized analysis Gilchrist et al. (1994) showed that valley incision could account for at most about 25 per cent of the elevation of mountain tops. Whereas these previous estimates of the effect of valley incision on the height of mountain peaks used analyses of topographic cross-sections, Small and Anderson (1998) used digital topography to estimate the predicted isostatic rebound from late Cenozoic valley incision into a low-relief upland in four mountain ranges in the western United States. They found that although late Cenozoic valley deepening could have raised the original surface by several hundred metres, erosion rates on the upland
were sufficient to preclude significant surface uplift of the peaks. Here we further examine the use of geographical information system (GIS) techniques to generalize the limiting analysis for the effect of valley development, and explore the application of this approach to the Olympic Mountains, Washington.

TWO-DIMENSIONAL MODEL AND CONSTRAINTS

Evaluation of the net volume of material removed from below mountain peaks requires identification of surfaces from between which to evaluate the volume of missing material. Other than for the simple problem of a palaeoerosion surface discussed above, the tops of current peaks define a logical reference surface, as erosion of anything above the present peaks only contributed to net lowering of the whole landscape. Such a surface connecting contemporary peaks need not represent an assumed palaeosurface. Instead it defines a reference surface for calculating the net differential erosion reflected in incision of the intervening valleys. In most mountain ranges, local exhumation and rock uplift will have varied both spatially and through time. However, analysis of the net effect of valley development only examines the cumulative local differences in ridgetop and valley-bottom erosion that generated local relief, as this is the only part of erosion that could have contributed to elevating mountain peaks. Similarly, the timing and rate of exhumation do not influence the net isostatic effect that can be constrained from contemporary topography. In contrast, evaluating the isostatic surface uplift over a particular time frame (i.e. to estimate a rate) requires knowledge of a well dated initial condition, such as an erosion surface. The second relevant surface is that defined by the present topography. Isostatic compensation of the volume of material missing from between these two surfaces represents the maximum possible influence of the cumulative effect of valley deepening over the course of uplift of a mountain range. In effect, the ‘superelevation’ of the peaks that may be calculated in this manner represents the elevation difference between the present peak heights and the land surface if there were no valleys. This superelevation therefore represents that proportion of the total elevation of the current peaks that could be due to the progressive effect of the differential erosion that carved valleys.

Assuming simple, periodic topography with uniform slopes (Figure 2) the cross-sectional area \( A_e \) of material eroded from below peaks to form a valley is given by:

\[
A_e = EH
\]

where \( E \) is the local elevation difference between the peaks and valley bottom (i.e. the local relief) and \( H \) is the hillslope length from ridgetop to valley bottom. In this simple cross-sectional view, the area of rock that isostatically compensates for the eroded material is given by:

\[
C[(\rho_c/\rho_m)EH] \tag{2}
\]

where \( C \) is the proportion of Airy isostatic compensation, which is \(< 1\) due to flexural rigidity of the crust, and \( \rho_c \) and \( \rho_m \) are the average densities of crustal and mantle material. Dividing the area of ‘missing’ material given by Equation 2 by the distance between peaks \( (2H) \) yields the average vertical rebound \( (\Delta E) \):

\[
\Delta E = C[(\rho_c/\rho_m)E/2] \tag{3}
\]

Assuming an average crustal density of 2800 kg m\(^{-3}\) and an average mantle density of 3300 kg m\(^{-3}\), Equation 3 indicates that at most (i.e. for \( C = 1 \)) 42 per cent of local relief translates into higher peaks for the case of uniform, periodic topography with narrow valley bottoms. The incision of wide valley bottoms increases the percentage of the local relief that could contribute to the surface uplift of mountain peaks (Figure 3). The cross-sectional area of material ‘missing’ from periodic topography with wide valley bottoms is given by:

\[
A_e = E(W + H) \tag{4}
\]
where $W$ is the width of the valley bottom. The vertical dimension of the compensating area of rock (i.e. the associated rock uplift) is given by:

$$E^c = \frac{m^\dagger E^\dagger W^\dagger}{W^\dagger 2^\dagger H^\dagger}$$

which indicates that the total proportion of the local relief translated into higher mountain tops varies with the width of the valley bottom. Using the average crustal and mantle densities discussed above, the maximum possible local increase in the height of mountain peaks attributable to valley incision (i.e. for $C = 1$) increases asymptotically from $(\rho_c/\rho_m)E/2$ for narrow valleys toward $(\rho_c/\rho_m)E$ at infinitely wide valley bottoms (Figure 4).

For the more general case of a variably incised landscape, the spatially averaged thickness of material missing from between peaks ($D_e$) may be expressed as:

$$D_e = A_e/W$$

and the compensating rise in the elevation of mountain peaks is given by:

$$\Delta E = C(\rho_c/\rho_m)(A_e/W)$$

where $A_e$ is the cross-sectional area of material missing from between peaks across a width $W$. Note, however, that three-dimensional incision of a landscape may result in greater local uplift than for these simple cross-sectional cases. For the three-dimensional case:

$$D_e = V_e/A$$
and \( \Delta E \) is given by:

\[
\Delta E = C \left( \frac{\rho_c}{\rho_m} \right) \left( \frac{V_e}{A} \right)
\]

where \( V_e \) is the volume of missing material over an area \( A \).

What does such an analysis show? The cumulative effect of valley incision on the height of peaks reveals nothing about either total denudation, as many kilometres of material may have been eroded from above the peaks, or paths of tectonic rock uplift, as there are many possible patterns of rock uplift and erosion rates that could account for that portion of the height of present topography that is not accounted for by valley geometry. The analysis described above accounts only for the net local differences between rock uplift and denudation that contributed to valley development. This approach estimates the maximum potential effect of valley enlargement on the height of mountain peaks because flexural rigidity of the crust may preclude full isostatic compensation, especially for small ranges that may be flexurally supported. Furthermore, other processes such as tectonic denudation, development of structural valleys and tectonic underplating (e.g. Forsyth, 1985) may reduce uplift from isostatic compensation. Due to such factors, the method of calculating isostatic adjustment described above simply yields a way to constrain the maximum possible contribution of valley development to the height of the surrounding peaks.

**APPLICATION TO THE OLYMPIC MOUNTAINS**

The Olympic Mountains are the topographic signature of a bend in the subduction zone that defines the Cascadia convergent margin (Brandon and Calderwood, 1990). The range first emerged above sea level in the middle Miocene and apatite fission track ages indicate relatively constant long-term exhumation rates of about 0.3 mm a\(^{-1}\) for the range as a whole and up to 0.75 mm a\(^{-1}\) in the core of the range (Brandon et al., 1998). The eastern end of the range is underlain by Eocene basalt and fore-arc basin strata, whereas the central and western portions of the range consist of Eocene to Miocene marine turbidites (Tabor and Cady, 1978). Prevailing winds blow from the west and there are strong gradients to annual rainfall across the range. Considering these factors, the Olympic mountains provide an interesting opportunity to study erosional and tectonic controls on the elevation of mountain peaks, as Quaternary glaciations scoured deep valleys into the heart of the range and subduction-related tectonic convergence, strong lithological contrasts, and a dramatic rainshadow impart a general asymmetry to the range.

Figure 4. Relation between the ratio of the valley bottom width to the hillslope length \((W/H)\) and the proportion of the local relief \((E)\) that translates into higher mountain top elevations for \( C = 1, \) and \( \rho_c/\rho_m = 0.84 \)

For our analysis we compiled a 30 m grid digital elevation model (DEM) of the Olympic Mountains from USGS 7.5’ quadrangle DEMs (Figure 5). Our approach involved fitting a smoothed surface to the elevations of local peaks to create a reference surface against which to evaluate the amount of material removed from between mountain tops (i.e. the amount of material differentially eroded due to valley development). To determine the appropriate surface for our analysis we fitted a series of surfaces to the envelope defined by the highest peaks within a search radius that varied from 1 to 15 km in 0.3 km increments. The total volume of material within the fitted surface was greatest with a search radius of between 5 and 10 km, and for our analysis we sought a maximum constraint on the ultimate effect on mountain heights. Hence, we used a 6-6 km search radius as it maximized the total volume beneath the fitted surface. The net volume of material differentially eroded from valleys was determined from the elevation difference for each 30 m grid cell between the DEM and the smoothed surface fitted to the peaks. Mean elevation, maximum elevation and the equivalent thickness of material differentially eroded by valley development (i.e. the volume of ‘missing’ material divided by grid area) were calculated for a 5 km grid from the respective values from the 30 m grid.
Figure 6. Representations of the Olympic Mountains showing the equivalent thickness of material eroded from below the fitted surface within each 5 km grid cell (top) and the maximum elevation within 5 km grid cells (bottom)
The spatial distribution of mean and maximum elevations in the Olympic Mountains exhibits some significant differences. The mean elevation generally increases to the east across the range. At a 5 km grid size, mean elevations in the western Olympics are below 1000 m; the central core of the range has mean elevations that range from 500 to 1500 m; and the eastern end of the range has mean elevations in the 1000 to 2000 m range. In contrast, the maximum elevations show two distinct areas of high peaks: a small area centred on 2417 m high Mount Olympus and a broader area of high peaks at the eastern end of the range (Figure 6a). Perhaps most striking is that the highest peak in the range does not sit in the area of greatest mean elevation—Olympus stands alone in the centre of the range.

The equivalent thickness of material differentially eroded from between mountain peaks (i.e. the net amount of ‘missing’ material) is greatest in the deeply incised Hoh, Queets and Elwha valleys which surround Mount Olympus (Figure 6b). In contrast, the area of high peaks at the eastern end of the range hosts relatively small valleys. Although flexural rigidity in the crust limits the amount of actual rebound to less than predicted for Airy isostasy, as discussed further below, these observations indicate that incision of deep valleys in the core of the range could have substantially affected the height of the highest peak.

Examination of trends in mean and maximum elevations across the Olympic Mountains shows that Mount Olympus rises above the general trend in mean elevation (Figure 7). Mean elevations across the range are asymmetric, rising gently from the Pacific Ocean to the core of the range and falling off rapidly on the eastern end of the range. Maximum elevations follow the general trend in mean elevation until the vicinity of Mount Olympus, which abruptly rises 500 to 600 m above the general trend. The cross-section shown in Figure 7 also illustrates the great size of the deep Elwha valley which crosses the profile immediately east of Mount Olympus. The valley of the Elwha River is comparable in size to the west-flowing Hoh and Queets River valleys, which respectively lie to the north and south of Mount Olympus.

A crude way to estimate the erosional rebound at Mount Olympus is to assume that the difference between the maximum elevations in the central and eastern portions of the range should be comparable to the differences in mean elevation and that any discrepancy is due to rebound from erosional unloading. Mount Olympus is 49 m higher than the highest point in the eastern portion of the range, even though the mean

Figure 7. Minimum, mean and maximum elevations along a west–east transect across the Olympic Mountains
elevations in the central Olympics are about 500 m lower (Figure 7). If this excess height of Mount Olympus over the general range of mean elevations in the core of the range reflects rebound due to erosion of the valleys that surround it, then the total rebound is roughly 550 m, or 23 per cent of the elevation of Mount Olympus.

THREE-DIMENSIONAL REBOUND MODEL

A more sophisticated model of the contribution of erosional unloading to the elevation of Mount Olympus can be obtained by incorporating the effect of flexural rigidity of the crust into predictions of the isostatic rock uplift in response to the negative load associated with valley development. Anderson (1994) modelled the three-dimensional flexural support for topography by summing the local deflection predicted by a model of response to point loads at each grid cell in a landscape. Here we invert this approach to predict the isostatic rebound from erosional unloading. Our analysis represents the vertical deflection due to a point load \( w \) by (Lambeck, 1988):

\[
w \left( \frac{r}{\alpha} \right) \left( \frac{q}{2\pi \rho_m g \alpha^2} \right) Kel \left( \frac{r}{\alpha} \right)
\]

(10)

where \( \frac{r}{\alpha} \) is the non-dimensional distance from the point load, \( \rho_m \) is the density of the mantle, \( q \) is the point load taken to be \(-z \rho_c \) dx dy, \( z \) is the equivalent eroded thickness, \( \rho_c \) is the density of the crust, dx and dy are the grid cell dimensions, \( Kel \) is the Kelvin function (see Abramowitz and Stegun, 1965), and the flexural parameter \( \alpha \) is given by:

\[
\alpha = (D/\rho_c g)^{1/4}
\]

(11)

where \( D \) is the flexural rigidity of the crust, which can be estimated from the effective elastic thickness of the crust through:

\[
D = ET^3/12(1 - v^2)
\]

(12)

where \( E \) is the modulus of elasticity, \( T \) is the effective elastic thickness, and \( v \) is Poisson’s ratio. In general \( E = 8.35 \times 10^{10} \) N m\(^{-2}\) and \( v = 0.25 \), but values for \( T \), and therefore \( D \), are more difficult to constrain.
The pattern of predicted isostatic rebound for the Olympic Mountains was calculated using Equation 10 to generate the expected pattern of compensation for the material eroded from below the fitted surface for a 300 m grid aggregated from values for each 30 m grid cell. This resulted in a predicted rebound pattern associated with each 300 m grid cell. The pattern of total net rebound was calculated by summing the rebound fields predicted from each of these patterns for each of these grid cells. The simulated pattern of isostatic response to valley development predicts the greatest amount of rebound in the vicinity of Mount Olympus, and little net rebound for the area of high peaks in the eastern end of the Olympics. The magnitude of erosional rebound varies with the effective elastic thickness of the crust. With a range of flexural rigidity of $10^{21}$ to $10^{23}$ N m, which corresponds to a range of $T$ values of 5 to 24 km, the predicted rebound at Mount Olympus ranges from 310 to 750 m (Figure 8).

DISCUSSION

The value assumed for the effective elastic thickness of the crust controls the magnitude of the predicted rebound of Mount Olympus. The northwest coast of the United States has an average crustal thickness of 20 km (Couch and Riddihough, 1989; Mooney and Weaver, 1989), and the crustal rocks that form the Olympic Mountains increase from a thickness of approximately 10 km at the coast to 30 km at their eastern margin (Brandon and Calderwood, 1990; Finn, 1990). Riddihough (1978) also favoured a crustal thickness of about 20 km beneath the Olympic Peninsula. However, the effective elastic thickness of the crust is likely to be much less due to the shallow hot slab. Hence, the effective elastic thickness beneath the Olympic Mountains is probably less than 20 km. Banks et al. (1977) concluded that the flexural rigidity of the crust in the United States was between $10^{21}$ and $10^{22}$ N m, and hence that the effective elastic thickness was between 5 and 11 km. Bechtel et al. (1990) also reported effective elastic thickness values < 10 km for western North America. We conclude that a value of $5 \leq T \leq 10$ km is reasonable for the Olympics. Hence, it is likely that roughly 500 to 750 m of the elevation of Mount Olympus is due to erosional unloading in response to valley development over the life of the range. This amount of ‘superelevation’ of the peak of Olympus was enough to raise it above peaks in the eastern end of the range.

The deep valleys that surround Mount Olympus were probably enlarged by valley glaciers during the Pleistocene. The addition of substantial elevation to Mount Olympus would have exerted a positive feedback on glacier development and the associated valley size in a manner described as a ‘topographic lightning rod’ by Brozovic et al. (1997). The prevailing winds blow from the west in the Olympics and there is a strong rainfall gradient across the range. Hence, as Olympus rose it would have intercepted a greater proportion of the incoming precipitation, which would have enhanced glacier growth, accelerated valley incision, and increased erosionally induced uplift of the remaining massif. Whether or not a strong feedback occurred, our analysis reveals that incision of deep valleys in the centre of the range is probably responsible for the location of the highest peak significantly westward of the area of greatest mean elevations.

It is also curious that long-term denudation rates are highest in the middle of the range (Brandon et al., 1998), whereas mean and peak elevations (except for Olympus) are higher at the eastern end of the range (Figure 7). Conventional wisdom holds that erosion rates increase with relief (Ruxton and McDougall, 1967; Ahnert, 1970), but the strong gradient in precipitation across the range probably retards erosion rates in the high topography of the eastern Olympics, as does the belt of highly competent basalt in this area. Similarity in mean slopes across the core of the Olympics (Figure 9) suggests that the centre of the range may be a threshold landscape, in which slopes are as steep as the rock can support (Schmidt and Montgomery, 1995; Burbank et al., 1996). If this is the case, then the high rates of exhumation in the core of the range may simply reflect the higher rainfall and generally steeper slopes than in the outer areas of the range.

The proportion of the total elevation of Mount Olympus that we estimate could be accounted for by erosional unloading (21 to 32 per cent) is comparable to the maximum estimates of 25 to 30 per cent in other regions based on cross-sectional analyses (Montgomery, 1994; Gilchrist et al., 1994). Appropriate estimates of the effective elastic thickness of the crust may be difficult to constrain, but it is generally recognized that valley incision will exert a greater effect on the height of mountain peaks in areas with weak crust and wide valleys than in areas with strong crust and narrow valleys. It is important to recognize that the approach
developed above constrains neither the total erosion-induced rock uplift nor the interaction with tectonically induced rock uplift. Neither can it evaluate the contribution of a discrete episode of valley enlargement to rates of surface uplift for mountain tops. What it can constrain, subject to appropriate caveats, is the degree to which isostatic response to valley development could have influenced the total elevation of mountain peaks. Additional range-specific geochronologic studies, such as that by Small and Anderson (1998), are needed to establish the magnitude of Quaternary valley deepening and establish the degree to which valley incision contributed to late Cenozoic mountain uplift.

ACKNOWLEDGEMENTS

We thank Kelin Whipple for an engaging correspondence, Doug Burbank for his critique of an earlier manuscript, and Mike Kirkby and an anonymous reviewer for their constructive suggestions.

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